FINNED CYLINDER IN SUPERSONIC FLOW

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UDC 629.782.015.3.025.1

An exact solution is obtained for the design regime of a fin assembly with a star-shaped cross section on a circular cylinder. We construct the solution with the aid of the stream surfaces downstream of plane compression shocks.

A technique was proposed in [1] for constructing pointed pyramidal bodies with a star-shaped cross section with the aid of the stream surfaces downstream of plane compression shocks. The technique can be extended to the case when the front part of the body is a cylindrical stream surface of a uniform (undisturbed) supersonic flow. Any combinations of plane shocks that yield the variations of the longitudinal and transverse body contours can be used.

From the line of intersection of the plane shock with this cylindrical surface there also emanates a cylindrical surface that is formed by the streamlines behind the shock, which mates at the ends of the arc of intersection with the planes of two wedges, the leading edges of which are the lines of intersection of the given shock with the two neighboring shocks.

In order that the cross section contour be continuous, the angles φ between the planes that are normal to the neighboring shocks and pass through the direction of the undisturbed flow velocity and the plane passing through this same direction and the edge of the wedge must satisfy the condition

$$\frac{\cos\varphi_{i+1}}{\cos\varphi_i}=\frac{\operatorname{tg}\theta_{i+1}}{\operatorname{tg}\theta_i}$$

(θ is the shock inclination angle). The contour of the fin assembly with star-shaped cross section that is obtained in this way will be closed if the sum of all the angles φ is equal to 2π. Intersection of the streamlines behind the neighboring shocks is avoided if the angle between their planes is less than π; this is the condition of the formation of a wedge with a finite apex angle. If all the plane shocks form a symmetric pyramid, the front part of the surface is also symmetric, and the axes coincide, then only drag will act on the fin assembly. If symmetry does not exist in one plane, then a moment relative to the longitudinal axis will also act on the fin assembly. This technique is of interest, since it makes it possible to obtain a fin assembly of different form, for which the force and moment are easily and accurately calculated from the pressure behind the shocks. The base pressure behind wedges [2] and the friction drag can be calculated quite simply.

We shall consider as an example a circular cylinder at zero angle of attack; this may be either a quite long cylinder or a cylinder with through flow, the flow nonuniformity near which is limited by thin entropy and boundary layers and can be neglected. From the line of intersection of the plane shock with the cylinder AB (Fig. 1) there emanates the cylindrical surface ABDC, formed by the streamlines behind the shock, having the inclination angle δ. In the section that is normal to the cylinder axis and is located at the distance ℓ from the end A of the line of intersection, specified by the angle φ, we write the coordinates of the contour of this surface (taking the cylinder radius to be unity) in the form

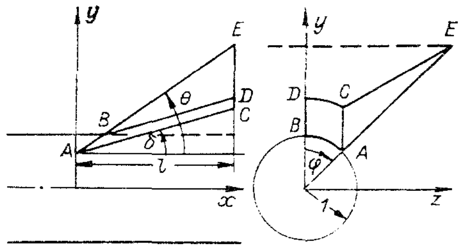

Fig. 1

Zhukovskii. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 4, pp. 94-96, July-August, 1993. Original article submitted July 27, 1992.

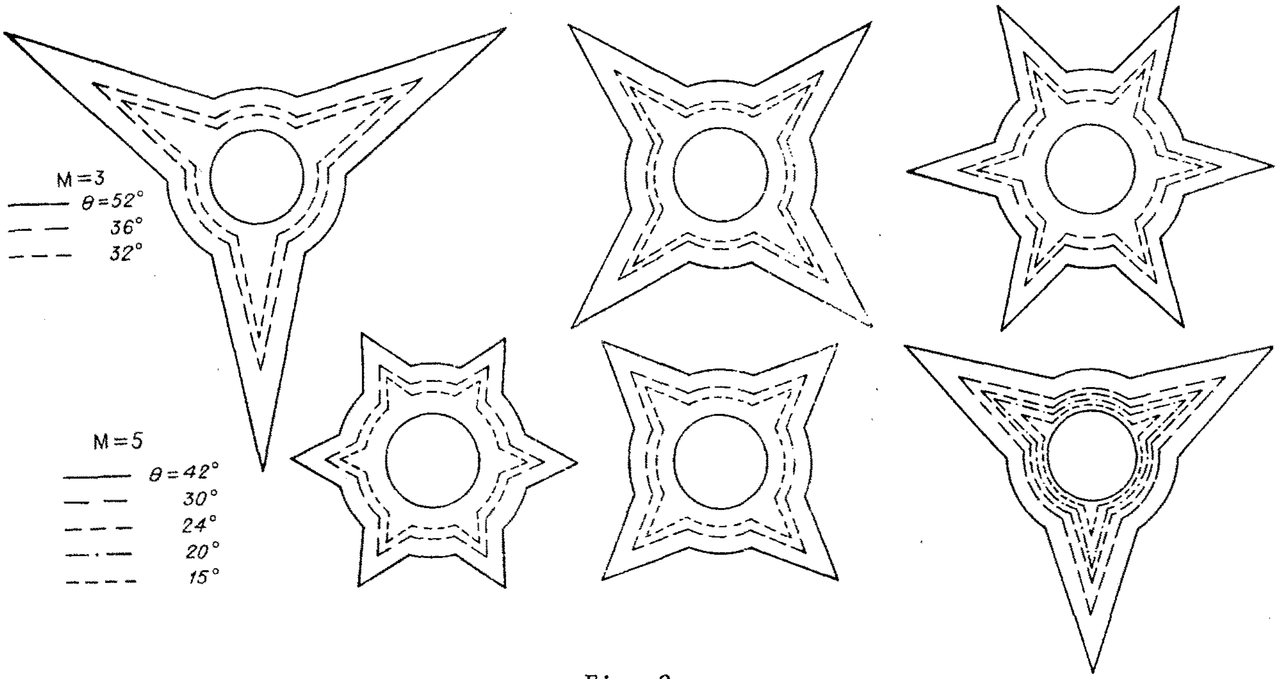

Fig. 2

$$y = \cos\psi\left(1 - \frac{\operatorname{tg}\delta}{\operatorname{tg}\theta}\right) + \left(l + \frac{\cos\varphi}{\operatorname{tg}\theta}\right)\operatorname{tg}\delta, \quad z = \sin\psi,$$

where ψ is the running angle, measured from the y axis. The line AE is the leading edge of the wedge (line of intersection of the neighboring shocks), the coordinates of the point E are:

$$y = \frac{z}{\operatorname{tg}\varphi} = \cos\varphi + l\operatorname{tg}\theta.$$

For calculation of the force and moment acting on the entire surface ABDCEA, it is sufficient to find its projection areas:

$$\sigma_y = \frac{\sigma_x}{\operatorname{tg}\delta} = \frac{\operatorname{tg}\varphi\,(l\ \operatorname{tg}\theta + \cos\varphi)^2 - \varphi}{2\operatorname{tg}\theta},$$

$$\sigma_z = (1 - \cos\varphi)\left(1 - \frac{\operatorname{tg}\delta}{\operatorname{tg}\theta}\right)\left(l - \frac{1 - \cos\varphi}{2\operatorname{tg}\theta}\right), \quad l \geqslant \frac{1 - \cos\varphi}{\operatorname{tg}\theta}.$$

Examples of fin assembly cross sections with different number of wedges for $\ell = 2$ are shown in Fig. 2. We note that if $\ell < (1 - \cos\varphi)/\tan\theta$, then the cylindrical surface behind the shock begins from the angle $\psi = \arccos(\cos\varphi + \ell\tan\theta)$. Conical [3] and intersecting plane [4] compression shocks can also be used for the construction of such bodies.

The author wishes to thank A. N. Minailos and K. P. Petrov for suggestions leading to the present study.

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